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Utilizing a Collaborative Cross Number Puzzle Game to Develop the Computing Ability of Addition and Subtraction

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ABSTRACT

While addition and subtraction is a key mathematical skill for young children, a typical activity for them in classrooms involves doing repetitive arithmetic calculation exercises. In this study, we explore a collaborative way for students to learn these skills in a technology-enabled way with wireless computers. Two classes, comprising a total of 52 students in Grade 4 (ages 10 or 11) participated in the study. They used the Group Scribbles software to run an adapted version of the "Cross Number Puzzle" that was designed with the "feedback" mechanism to assist students' problem solving. In one class, students played the game individually and in the other class, students played the game collaboratively. The low-ability students in the collaborative class were found to have made the most significant progress in arithmetic skills through playing this game. Three dominant interactive collaboration patterns, one contributing to productive interactions and two to less productive interactions, were also identified in the students' collaboration.

Keywords

Computer-supported collaborative learning, feedback mechanism, Cross Number Puzzle, computing ability

Introduction

Computational skills in addition and subtraction are an important component in any mathematics curriculum for young children. In seeking to design more interesting learning experiences for children to learn these skills, we designed and implemented a learning activity which is an adapted version of the "Cross Number Puzzle" on a technology platform. The aim of this game is to promote the flexible application of addition and subtraction skills, and to enhance children's capacity to build up their arithmetic skills progressively. We conducted a study on two classes of students to explore different collaborative learning patterns that involved students working together on solving arithmetic problems. We also examined differences between individual learning and collaborative learning.

A variety of educators have classified operating addition and subtraction story problems into four problem types: change, combine, compare and equalizer (Carpenter, Hiebrt, and Moser, 1981; Fuson, 1992; Gustein and Romberg, 1995). English (1998) points out that change and combine are easier while take-away and compare are more difficult challenges for elementary school students. In an arithmetic equation, any of the three numbers could be the unknown number. We adopted this widely used method in our study. Fuson (1992) defines these three types of "change" (placeholder) as: Missing End, Missing Change, and Missing Start. Van de Walle (2001) also classifies the type of "change" into three types: result-unknown, change- unknown and initial-unknown. These three types of problems present different levels of difficulty to the students. In Tompson's study (1983), the initial unknown is most difficult. If the student applies the direct modeling strategy by using counters or tally marks to model directly the action or relationships described in the problem (Carpenter et al., 1993), he or she always does not know how many counters to be put down to begin with. Table 1 illustrates the three levels of change types in story problems. Level I is when the result number is unknown, Level II is when the change number is unknown, and Level III is when the initial number is unknown (Carey 1991; Peterson et al, 1991).

With these three levels in Table 1 in mind, we designed our system to incorporate five stages of problem posing to the students (Table 2):

In stage 1, the student is required to derive the answer of an arithmetic expression (result number unknown), inculcating the skills of basic addition and subtraction, for example: $3 \pm 2 = \square$.

In stage 2, the arithmetic operator is removed. Students were required to understand the concept of arithmetic operator, for example: $3 \square 2 = 5$.

In stage 3, the change amount is removed, for example: $3 \pm \square = 5$.

In stage 4, the initial amount is removed, for example: □ ± 3 = 5.
In stage 5, both the initial and the change numbers are removed. It is more difficult as the sentence includes two variables: change amount and initial amount unknown. For example: □ ± □ = 5.

Table 1. Three levels of “Change” types in story problems

Change Types	Join(Add to)	Separate(Take from)
Result number unknown	John has 5 cookies; Marry gave 5 cookies. How many does John have now? Standard sentence: A + B = □	John has 8 cookies; he gave Marry 5 cookies. How many does John have now? Standard sentence: A – B = □
Change number unknown	John has 5 cookies; Marry gave some cookies, and now John has 8. How many did John get from Marry? Standard sentence: A + □ = B	John has 8 cookies; he gave Marry some cookies, and now John has 5. How many did John give to Marry? Standard sentence: A - □ = B
Initial number unknown	John has some cookies; Marry gave 5 cookies, and now John has 8. How many cookies did John has before Marry gave him some? Standard sentence: □ + A = B	John has some cookies; he gave 5 cookies to Marry, and now John has 3. How many cookies did John has before he gave some to Marry? Standard sentence: □ – A = B

Table 2. Level of difficulty in the design

Level of difficulty	Description	Example
Level 1	Result number unknown — basic skill practice	A ± B =□
Level 2	Remove operator—between basic skill practice and comprehension application	A □ B = C
Level 3	Change number unknown add-to or subtraction—comprehension application	A ± □ = B
Level 4	Initial number unknown add-to and subtraction—comprehension application	□ ± A = B
Level 5	Change number unknown and Initial number unknown, addend or summand type —the most difficult level	□ ± □ = A

Design of the “Feedback” system

A ‘feedback’ mechanism was introduced to the game design in this study. Feedback is considered to have strong impact on the learning process and result (Kulhavy and Stock, 1989; Bangert-Drowns, Kulick, Kulik and Morgan, 1991; Balacheff and Kaput 1996). Appropriate feedback can lead learners to focus on key elements of learning. They can always adjust their learning strategies to try to close the gap between their actual performance and the learning goals. They reflect on their learning by a self-monitoring feedback loop. Hence, they can change their learning strategies in the follow-up learning and seek a better way of learning (Alexander and Shin, 2000). The learner understands the gap or difference between her own concept and the target concept, and the perceived difference can provide information for her to amend her ideas and knowledge, thus enacting a process of signal feedback.

Schmidt (1991) proposes that feedback is the result of a series of actions. It represents the personal response or reaction to the information they received. The feedback itself is a problem solving process that checks the performance of action to improve a person or a group. In technology-enabled learning, feedback is typically provided as messages shown to students after their responses (Cohen, 1985). Siedentop (1991) points out that feedback can promote the interaction between the teacher and the learners. Teachers can provide feedback to students in terms of their actions and performance, which enable them to know or to amend their understandings and may boost their enthusiasm for learning (Keh, 1992).

There are three forms of feedback: immediate feedback, summary feedback and compromise feedback (Schmidt and Wrisberg, 2000). Collins, Carnine, and Gersten (1987) point out three levels of feedback messages: (1) little feedback: just show the answer is right or wrong; (2) basic feedback: if answer is not correct then show right answer; and (3) descriptive feedback: give some hints to learner to help her to obtain the right answer. Descriptive feedback can increase the motivation for learners to try or solve new tasks and new problems. The feedback mechanism provided by software systems mainly involves five levels such as: no feedback, knowledge of response, knowledge of correct response, answer until correct, and elaboration feedback, as summarized by Sales (1998). In our research, the feedback mechanism is designed as follows (Figure 1):

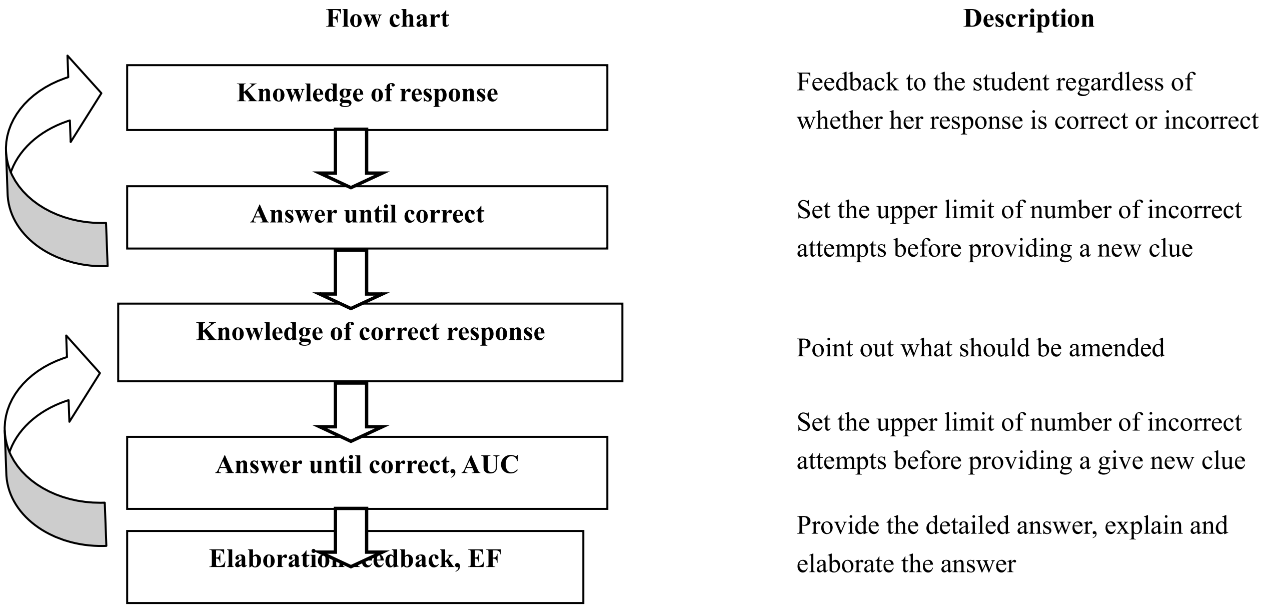

Figure 1. Five-level feedback mechanism

Progressive feedback is provided in a step by step manner in this five-level mechanism. In the 'knowledge of response', the student get a response whatever she replies correctly. Then a constraint on the upper limit of incorrect attempts is invoked to allow some trial and error. If the student fails after a number of trials, a clue or a suggestion would be offered to her for another round of trial and error. In the 'elaboration feedback', the correct answer together with detailed explanations will be provided.

Methodology

Fifty two students in Grade 4 (ages 10 or 11) participated in our study. They learned some basic addition and subtraction since Grade 1 but they needed to connect this to the new concepts and skills required for Grade 4 mathematics. In this research we explored the effects of "Cross number puzzle" game applied in learning, which was designed to provide the feedback mechanism. We had two experimental classes: students in Class A played the "Cross number puzzle" game in small groups, and students in Class B played the game individually. All students were grouped according to their average scores of the previous three tests in this term. Using percentile ranking, those students with the percentile rank of score over 73% were classified as high-math achievers; those with percentile rank of score between 27% and 72% were classified as medium-math achievers, and those with percentile below 26% were classified as low math achievement. Students in class A were divided into homogeneous groups with three per group. 6 students in the high-achiever group form two groups. Three medium-achiever groups comprised 9 students and another three low-achiever groups comprised 9 students.

We utilized Group Scribbles (GS) as the platform for the game, and conducted analysis of the collaborative work within these groups. GS is a computer-supported collaborative learning system developed by SRI International to conduct small-group collaborative concept mapping activities (Chaudhury, et al, 2006; Looi, Chen and Ng, 2010). Each student has a Tablet PC which has a screen divided into upper and lower frames (Figure 2). The lower frame is for individual cognition, that is, the student sketches or types her answer individually. The upper frame is a shared space (public board) in which the students show all of their individual answers, and work together as a group. They can even check the work from other groups by clicking the button on the top right corner (see Figure 2). The teacher can monitor their process of learning and provide appropriate guidance.

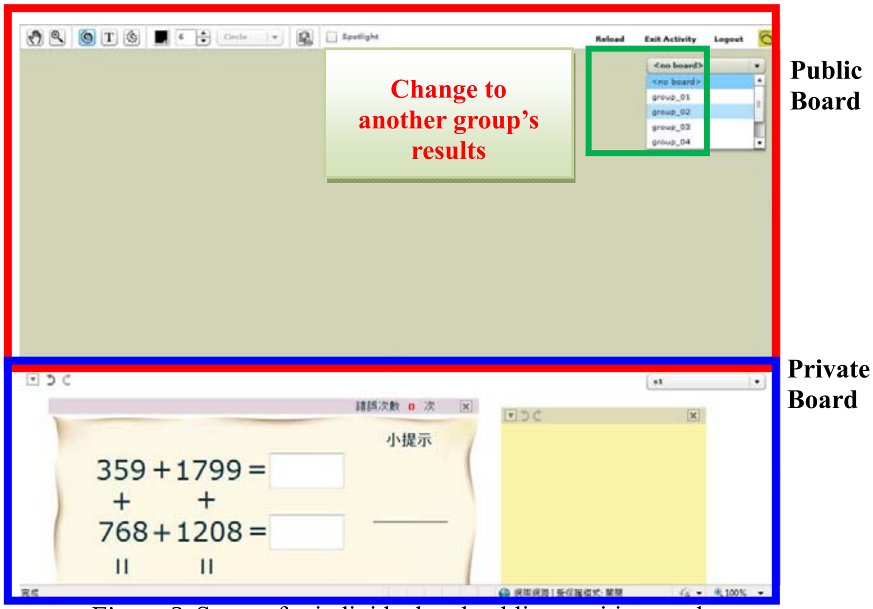

Figure 2. Spaces for individual and public cognition work

Figure 2 shows the interface of "Cross Number Puzzle" in GS. We designed questions ranging from the easy to the difficult in terms of the five levels of difficulty. When the students complete the calculation, they can fill in the answer box and press OK button under the question area to submit. If the answer is correct, there will be a brief description of the key points. If the answer is wrong, the system will execute a step-by-step tip based on the number of errors from the user inputs (Figure 8, a popup box with tips shown in individual area). The action repeats until the maximum number of errors reaches the upper limit. Then the system will show the correct answer and the methods of problem-solving. Four different types of questions were shown below (Figure 3 to Figure 6) in the "Cross Number Puzzle".

Figure 3. Question type 1

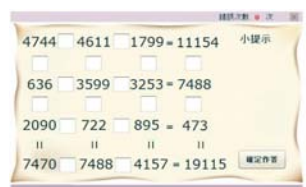

Figure 4. Question type 2

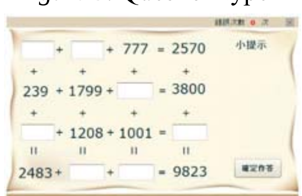

Figure 5. Question type 3

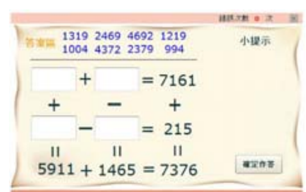

Figure 6. Question type 4

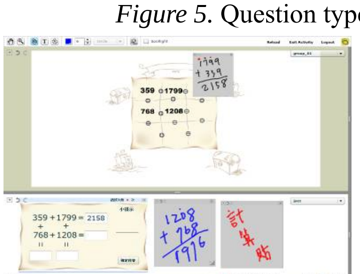

Figure 7. Interface of individual and public work

Figure 8. Tips

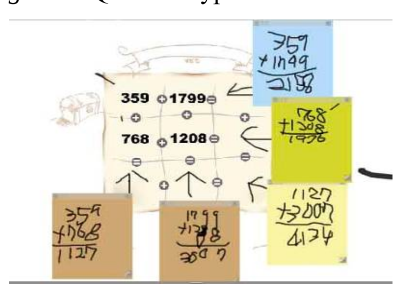

Figure 9. Calculation process

Figure 7 shows a screenshot of one student's calculation process. Questions appeared in the private board and the individuals could paste their sketches to the public board. Figure 9 provides the screenshot of the calculation process in a four-member group, in which each individual pasted his/her sketches using a different color.

The study for both classes lasted for four weeks. In the first week, a session for 30-minute pre-test and 20-minute training was executed. Students were asked to be familiarized with GS and the operation of the game with simple exercises. In the second week, the game was played in one lesson lasting for 60 minutes, followed by a 30- minute post-test and a 20-minute questionnaire in the third week. In the fourth week we interviewed the teachers and students. A pre-activity and three learning activities were included in this study.

In the game playing session, three tasks were designed and implemented in collaborative group (Class A) and individual group (Class B) separately. Students were asked to fill in the operator in an arithmetic equation in Activity 1. Activity 2 is about filling in the unknown number while in Activity 3, students were asked to estimate using trial-and-error methods to solve the problem. The only difference between these two classes is that students in Class B played the game individually but students in Class A played it collaboratively. Figure 10 below gives an example of a game screenshot of a collaborative group with four members. Group members could use their private boards for sketches and confirmatory calculations. Then they could post their sketches or results to the public board. Figure 11 shows an example of a student's work from a student who worked individually. We can inspect this student's solution path to identify and understand his working strategy.

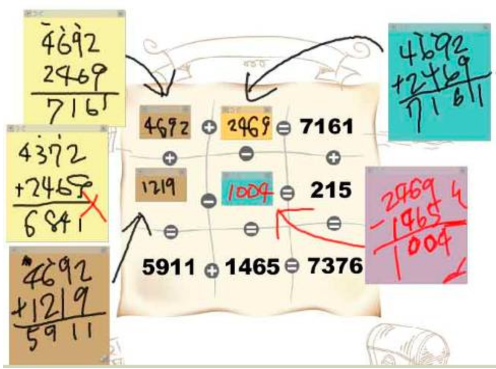

Figure 10. Change number unknown and initial number unknown exercise – a group's work

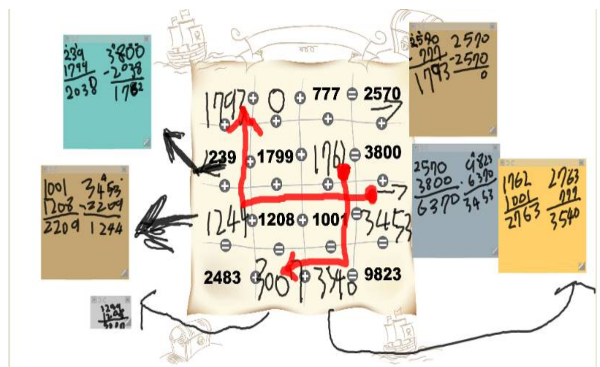

Figure 11. Change number unknown and initial unknown exercise – a student who worked individually

Findings

We analyzed students' scores in pre and post-tests, collected questionnaires, did video-recording of their activities in classes and tracked their screens in the process of game playing. The pre-test and post-test have the same questions but they are ordered differently. We also verify the item discrimination index and had the questions validated by a experienced mathematics teacher.

Results of the assessment of computing ability

We administered pre-tests and post-tests and performed independent sample t-test of two classes on their results.

Table 3 shows the pre and post test results of Class A. Students in Class A have higher average scores in the post-test. Their average score was increased by 13.00, from 50.29 in pre-test to 63.29 in post-test ($p=0.002<0.01$). This indicates students in Class A made greater progress than those students in Class B through playing the game collaboratively. Further observation of these collaborative groups suggests that the low math-achiever students made the most significant progress, which can be easily gathered from the following table.

Table 3. T-test of pre and post tests in collaborative group

Number of Participants (N=24)	t-test of collaborative group							
Tests	**Number of Participant**	**Min.**	**Max.**	**Average**	**SD**	**Progress**	**t**	**p**
Pre-test of high-achievers	6	63.34	76.68	70.56	3.141	7.22	1.308	.248
Post-test of high- achievers		48.34	100	77.78	10.576			
Pre-test of medium achievers	9	45.01	66.68	56.48	5.182	6.47	.880	.404
Post-test of medium-achievers		40.01	98.35	62.96	14.454			
Pre-test of low-achievers	9	6.67	55.01	30.56	10.111	23.33	4.834	**.001****
Post-test of low-achievers		16.67	76.68	53.89	9.874			

The low-achiever groups in Class A were found to have the highest increase in post-test scores at a high level of significance (P=0.001). This indicates that low-achievers of these collaborative groups derived the most benefits in this study.

Table 4 showed further analyses conducted on three different types of test questions on “addition and subtraction”. Students had better scores in all three types of questions in the post-test. But the low-achiever groups achieved significantly highest improvement in questions of “basic computing”, “unknown constant” and “Cross Number Puzzle” with the increase of average score 9.63, 7.38 and 6.32 respectively. This suggests that these low-achievers benefited the most from the “Cross Number Puzzle” in improving their basic arithmetic skills.

Table 4. Low achieved students’ progress in Pre and Post tests

Low achievers in Class A (N=9)		Pre -test	Post -test	Average increased scores	Ratio of progress in different questions
Basic computing skills	Score of question 1 to 5 (33.33)	18.52	28.15	9.63	41.3%
Unknow constant	Score of question 6 to12 (46.67)	8.90	16.28	7.38	31.7%
Cross number puzzle	Score of question 13 to 15 (20.00)	3.14	9.46	6.32	27.0%

On the other hand, the average score of Class B (the individual group) is 4.17 higher in the post-test (57.21) than in the pre-test (53.04) at a significant level of .026 ($p<0.05$). This indicates that learners in Class B also made progress through playing the game.

To gain further insight about the differences of improvement between the individual group and the collaborative group, regression analysis was done to judge the relationship between the scores of pretest (as the independent variable) and the score of post-test (as the dependent variable) within both classes. The results of F-test for pre-test ($F = 2.487$, p =. 121, p> .05) shows that the individual group (Class B) and the collaborative group (Class A) can be regarded homogeneous.

However, Table 5 below provides the result of F-test ($F = 4.479$, p =. 039, p <.05), which is significant at the p<.05 level. The striking result to emerge from the data is that the collaborative group had much greater improvement than the individual group in this study although they played the same game.

Table 5. ANOVA for individual group and collaborative group

Analysis of variance for Class A and Class B					
Item	Sum of squares	DF	Mean Square	F value	Sig.
Inter-group	313.995	1	313.995	4.479	.039*
In-group	3434.814	48	70.098		

Collaboration

Questionnaire results illustrated 85% students tried to do cooperation and discussion before they submitted the answer when they played the “Cross number puzzle” game. There was one high-achiever student who did not discuss with others when he did his calculations. He explained in the follow-up interview that he was quite confident and only shared his results with others when he completed all his calculations. 87.5% students claimed that it was much easier to complete the calculations with collaboration than doing them individually. Those students without confidence in mathematics found it easier to share their own ideas with others and complete the calculations together. All students agreed that they derived benefits from discussion with other classmates.

“Tips” usage in Class A and Class B

As we mentioned before, students in Class A play the game collaboratively in groups while students in Class B completed the game individually. We can easily conclude from Table 5 below that feedback in the form of “Tips” was much more frequently used in Class B than in Class A. It suggests that when a student encounters problems and difficulties and in the situation where there is no help from peers, he or she would search help from the “feedback” system. On the other hand, students in Class A would discuss their strategies to solve the problem within a group first, allocating cooperative work among group members. They only referred to the “feedback” system when every student in the group was uncertain on how to proceed They used the “tips” less often then students in Class B. Students in Class B seldom initiatively asked for help from their classmates but only referred to the tips when they encountered difficulties. However either in Class A or Class B, high-achiever students seemed to have used the “tips” far less than low-achiever students. Low achieving students relied more on “tips”.

Table 6. “Tips” usage in Class A and Class B

Number of use in different group	Class A (N=24)	Class B (N=28)
High-achiever	0.54	0.84
Medium-achiever	0.71	1.31
Low-achiever	1.25	2.09
Average usage	0.86	1.40

We analyze further students’ collaborative activities (in Class A) in the process of game playing which included three tasks to be completed in the game.

Pre-activity: Result number unknown

Before formal learning activities, a pre-activity was run with two objectives: familiarization with the GS system operation and warming-up the student work in doing arithmetic calculations. The question model is A ±B =□ which is the level one difficulty. 23 of 24 students in collaborative groups finished this activity.

Learning Activity 1: Remove the operator

Four different patterns of collaborative problem solving were found in their activities of “remove the operator”: whole-group-deciding, two-member-deciding, leader-deciding and individual deciding. Group 6 had the decision made by all group members. Three groups, Group 1, Group 3 and Group7 decided the answer on an individual basis. Two groups followed the two-member deciding pattern and the other rest two groups the leader-deciding pattern. The following figures (Figure 12 to Figure 15) shows different layout of the game in different collaborative methods. For example, in figure 11, three students in group 6 (one student in one color of “+”) posted their answer as 4777 +++ 4611 +++ 1799 = 11154, six “+” and one “=”. All these three students could perform the addition operation correctly. Therefore we could judge that this group’s answer was decided by the whole group. In Figure 14, there is only one answer being pasted, and when we referred to the video recording, we found this group was clearly one of leader-deciding.

Figure 12. Whole-group-deciding

Figure 13. Two-member-deciding

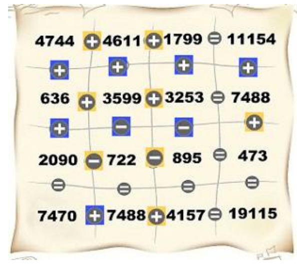

Figure 14. Leader-deciding

Figure 15. Individual deciding

Learning Activity 2: Fill in the figure in the formula sentence

To enable learners get the unknown number in the puzzle by observing, calculating those given numbers and estimating the result, for example , A ± □ = B & □ ± A = B, tasks division and coordination were necessary in one group. From the procedural layouts of the game on the screen we got some insights of methods of students' collaboration and their strategies to complete the calculation. The results were shown in Table 7.

Table 7. Methods of collaboration in Class A (8 groups)

Methods	Description	Group	Ratio of different method used
Individual calculation	Group members did the calculation by themselves individually. Little collaboration occurred.	G1, G2	25.0%
Comparison	Started from different thread and compare each other's result at the intersection	G3, G5	25.0%
Relay	One finish one section and another take over to continue calculating	G4	12.5%
Assisted calculation	One of the group members is in charge of all calculation and other members checking his/her calculating process	G6	12.5%
Through-out calculation	Some members calculate from the beginning to the end and other members calculate from the end to the beginning then they compare at the intersection.	G7, G8	25.0%

Learning Activity 3: Fill in Multi-unknown

This most difficult task (Level 5), students will fit in multiple unknowns equation like □+□=C or □-□=C. After analysis student's working path, trace group's problem solving strategies. The most frequently used collaboration strategy is cross calculation (about 50%). Table 8 shows the results.

Table 8. Methods of collaboration in Class A (8 groups)

Methods	Description	Group	Ratio of different methods used
Individual calculation	Group members did the calculation by themselves individually.	G1	12.5%
Cross calculation	Started from different paths and compare each other's result at the intersection	G2, G3, G4, G5	50.0%
Reverse calculation	Some members calculated following vertical or horizontal paths, other members calculated from the result to get the answers.	G6, G7, G8	37.5%

Interactive collaboration patterns

By analyzing the video recordings of the students' activities together with the recording of computer screens in Class A, different interactive patterns in eight groups were identified. We will illustrate these interaction patterns in the following diagrams. The double-arrow ↔ represents the full dual interaction between two learners. For example, if student S1 communicates much with student S2, we mark it with S1 ↔ S2. The arrow → represents strong one-way interaction and the dashed arrow ⇢ represents weaker one-way interaction. For example, if student S5 tries to discuss with student S6, but S6 does not respond, we mark it with S5⇢S6. If student S4 talks often to student S3, but student S3 does not respond, we mark it with S4→S3.

Milson (1973) identified seven frequently occurring interactive patterns within small learning groups, namely:
(1) Unresponsive: The leader communicates with other members but they do not respond;
(2) Unsocial: There is no interaction among the group members;
(3) Dominant leader: The leader communicates with group members who revere and follow the leader;
(4) Tete-a-tete: Every member interacts with his/her neighbors, which may lead to emergent sub-groups;
(5) Fragmented, cliquish: Fragmented interactions within individual sub-groups but no interaction at the full-group level;
(6) Stilted: Each and every member interacts with his/her neighbors; the interactions have yet to reach an ideal status albeit some individuals have opportunities to interact with each others;
(7) Ideal: All the group members actively interact with each others, and there are multiple communicative paths.

Three of Milson’s interactive patterns were identified in our study.

The ideal interactions

As shown in Figure 16, Group 3, Group 4, Group 5, Group 6 and Group 8 had full dual communication when they had discussions. Before they started to do the calculations, they had a discussion and allocated work amongst the group members. Everyone in the group was engaged in the discussion and helped each other solve problems, and this alleviates any difficulties encountered by the students in system operations or calculations. Because of the successful implementation of the division of labor and good communication, these groups tended to have good performance of learning activities.

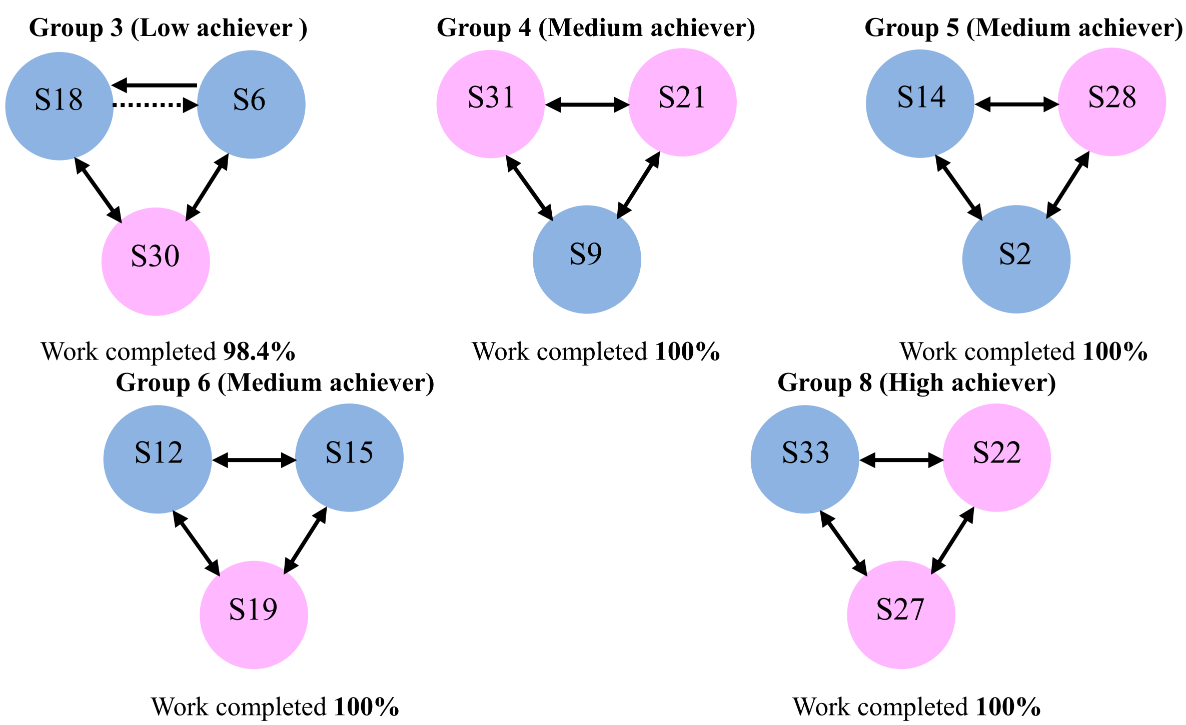

Figure16. Ideal interactive patterns in Group 3, 4, 5, 6 and 8

In Figure 16, the percent of work completed means total progress in all activities, for example, Group 3 had finished 100% questions in pre-activity, learning activities 2 and 3, but completed 93.75% for learning activity 1, so this group’s progress is 98.4% ((1+1+1+0.9375)/4). Group 8 finished all problems in all activities and got 100% progress.

The fragmented interactions

Group 2 and Group 7 manifested this interactive pattern, as illustrated in Figure 17. Analysis of the video recordings indicated that they did not have explicit cooperation at the beginning. Some members in these groups started individual calculations. They only discussed when they encountered difficulties and problems. Once the problem was solved, they went back to individual work. Some members of the group had weak communication or one-way communication with other members. At times a member of the group appeared isolated. Errors and omissions occurred due to their unsuccessful communication resulting in unproductive sharing.

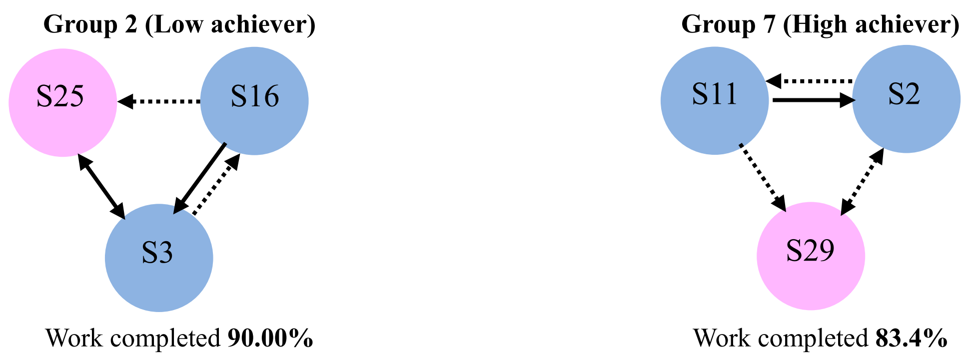

Figure 17. Fragmented interactions in Group 2 and Group 7

The unresponsive interactions

The interaction in Group 1 was unresponsive as shown in Figure 18. Video analysis shows that at the beginning one of the members was quite active. He tried to allocate the task to another group member. However both group members paid attention only to their individual calculation and neglected each other suggestions. This learner obtained little useful help from his group members and the whole group performed not well. They only finished 66.88% of the work.

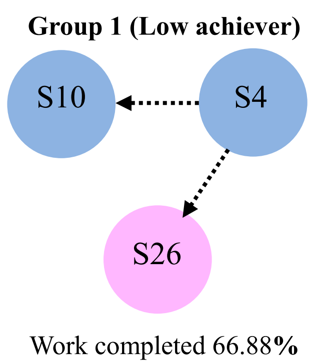

Figure 18. Unresponsive interactive pattern in Group 1

Teacher’s voices

The students of Class A were motivated and had good sharing in their group tasks. The teacher of Class A shared his experience with us in our interview with him:

"Most students were encouraged to have more discussions in this class. One of the high-achiever groups had conflicts during the discussion because everybody exhibited high confidence and expectation. Another medium-achieving group showed great enthusiasm in collaborative learning with one of them playing the role as a leader. "

"Every student in Class A could get feedback from the system as well as from other members. The consensus achieved in the group made the whole class improve. However in Class B, students had great diversity in their responses. Some students produced good responses when they understood but for some others, they were not sure and thus they talked with their neighbors. And some of them just immersed themselves in individual work and require the teacher to guide them when they encountered difficulties. "

Figure 19 to Figure 22 illustrate some of students' activities in the class.

Collaboration in groups

Figure 19. Students concentrating on their own work

Figure 20. Students helping each other

Figure 21. Students discussing problem-solving strategy

Figure 22. Students engaging in intense discussions

Conclusion and discussion

This paper investigated the benefits of learning addition and subtraction through the game "Cross Number Puzzle" on Group Scribbles. Our observations and investigations of the two classes who played the game individually and collaboratively respectively showed some interesting differences.

Effects of collaboration

The collaborative learning groups (Class A) were found to have made greater progress than individual learning groups (Class B). It suggests that collaborative learning may have enhanced learning effectiveness. From the statistics, we can conclude the low-achiever students benefited the most in this "cross number game". Collaboration also plays an important role in enhancing learning in Class A with the incorporation of the "feedback system" and collaboration strategies.

Benefits of feedback system

In both classes, the low-achiever students accessed the "tips" most often while the high-achiever the least. The individual learning groups in Class B had much higher frequency of access to "Tips". It indicates that the collaboration among group members in Class A did assist students' problem-solving. They relied less on the "feedback" system because they could get help from group members. However, in both classes, the low-achiever students had the highest demand for "tips" for help.

Methods of problem solving

Students in collaborative learning groups presented four different methods of problem solving in their activities of "removing the operator": whole-group-deciding, two-member-deciding, leader-deciding and individual deciding. In the activity of "fill in the figure in the expression," the students had five methods of calculations: individual calculation, comparison, relay, assisted calculation and through-out calculation. Students also showed four different ways of calculation: free calculation, calculate from the top, calculate from the bottom and calculate from both the top and bottom. They did the calculation in three different collaborative ways: each student calculates the whole thing him/herself; one student started from the top and the other started from the bottom; and they did backwards calculation for checking.

Interactive patterns

Three interactive patterns were found in this study. Among these, the ideal interaction occurred most often. The students all did well in their collaboration. The groups doing fragment and unresponsive interaction were not as interactive as the ideal groups. They had fewer communication and little cooperation.

Future work

Based on these findings in this study, we make the following recommendations for future research.

More experiments to further probe the "Cross Number Puzzle"

Our study has limitations of time and scale. To make the cross number puzzle more applicable, we may need to do more experiments and expand the number of users.

Larger shared screen

A big screen to display the public board for all group members could assist the discussion within a group by providing a focal point of attention. The individual board could still be retained in the screen of the students' personal Tablet PC for their private cognition.

Adaptive feedback

We only offered phased hints to students in this "Feedback system". The feedback only includes the general direction of calculation concept and the problem solving process. If system can diagnose and evaluate the individual student's errors, system can provide each student with the individual corresponding solutions or suggestions to fit his skills.

Incorporating a timer

From our analysis of the frequency of feedback in this study, a timer could be added to the system to record the duration of problem solving by each user. This would also enable the teacher to gauge the time used by the students at each stage of their problem-solving.

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