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Improving Pupils' Mathematical Communication Abilities through Computer-Supported Reciprocal Peer Tutoring

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ABSTRACT

This study examined how to foster pupils' mathematical communication abilities by using tablet PCs. Students were encouraged to generate math creations (including mathematical representation, solution, and solution explanation of word problems) as their teaching materials and reciprocally tutor classmates to increase opportunities for mathematical communication during a semester. A reciprocal peer-tutoring-enhanced mathematical communication system was designed for supporting students' math creations and reciprocal peer-tutoring activities. An experiment involving 51 second-graders was conducted to evaluate their improvement in mathematical communication abilities. While the control group received one-to-one self-learning mathematical materials and teacher-led instruction, the experimental group was engaged in computer-supported reciprocal peer tutoring in the same environment with the same materials. Both groups were evaluated by using a mathematical communication ability assessment. The result indicated that the experimental group outperformed the control group in the assessment. Additionally, math creations were analyzed for assessing students' formative development. The results showed that students' math creations became clearer and more efficient. In other words, their mathematical representations and solution explanations became more accurate after the learning activity.

Keywords

Mathematical communication ability, Reciprocal peer tutoring, Math creation

Introduction

Mathematical communication, a fundamental mathematics educational objective that involves cognitive and social activities (Baroody & Ginsburg, 1990), is used to engage students in communicative situations for increasing learning interaction with others to obtain mutual mathematical ideas (Silver & Smith, 1996), share mathematical thoughts, develop mathematical concepts and strategies, and reflect on their current mathematical understanding (Whitin & Whitin, 2000; Cooke & Buchholz, 2005). Mathematical communication abilities also include expressing mathematical thought by using mathematical language clearly, precisely, and succinctly (National Council of Teachers of Mathematics, 2000); understanding others' mathematical equations and concepts (Lin & Lee, 2004; Lin, Shann, & Lin, 2008); and evaluating others' mathematical concepts by, for example, asking meaningful questions and explaining the reasons for others' incorrect mathematical thought (Lin & Lee, 2004).

Various means of fostering mathematical communication abilities have been proposed. For example, Baroody and Ginsburg (1990) suggested that students should communicate mathematical ideas through representing, listening, discussing, reading, and writing. Cobb, Boufi, McClain, and Whitenack (1997) also claimed that students' mathematical discourse in classrooms can support their conceptual development, while Shimizu and Lambdin (1997) revealed that students who write about the thinking process of their solutions can organize complex thoughts and evaluate their own opinions. Similarly, Steele and Arth (1998) argued that reflecting on how to solve a problem by writing their solutions can facilitate students to explain their thinking process more clearly, thereby benefitting themselves in learning mathematics and learning to communicate mathematically by constructing mathematical artifacts as well as developing and evaluating mathematical arguments (National Council of Teachers of Mathematics, 2000). Similarly, students can adopt written mathematical communication by using text, figures, tables, pictures, diagrams, or mathematical symbols to provide critical evidence of their mathematics ideas and concepts (Mooney, Hansen, Ferrie, Fox, & Wrathmell, 2012; Whitin & Whitin, 2000). Additionally, students can learn mathematics by observing, interacting with, and manipulating physical objects and the representations of objects and

concepts (Sedig, 2008). Besides, students may support their claims by describing and recognizing patterns, generalizing rules, and using various types of representations (Moschkovich, 2012). These studies have demonstrated various methods for expressing or explaining mathematical ideas by writing or creating mathematical materials. The content of mathematical communication can be treated as an artifact of reflection, refinement, discussion, and modification. Therefore, mathematical communication ability should be fostered in students by simultaneously training their oral expression and various mathematical representations for explaining their understanding of mathematical ideas and strategies concretely as well as sharing their work with one another (Dacey & Eston, 2002).

To improve students' mathematical communication ability, several studies have shown the potential of using computers. For instance, Koedinger, Anderson, Hadley, and Mark (1997) built Practical Algebra Tutor (PAT) system to engage students in investigating real-world problems and using algebraic tools to generate multiple representations (tables, graphs, and symbols) for solving algebra problems and communicating results. Stahl (2009) designed a Virtual Math Team (VMT) system for students' exploring and discussing mathematical topics with peers for improving the quality of mathematical conception through online mathematical text-based chatting. Furthermore, Tsuei (2012) adopted G-Math, a synchronous peer-tutoring system, for pupils to discuss mathematical word problem solving via a sharing mechanism, which takes advantage of the availability of students' works to communication and obtain mutual perspectives. Additionally, Bruce, McPherson, Sabeti, and Flynn (2011) allowed students to express their mathematical thought and learn their strengths from peers' feedback during discussions.

Most importantly, the aforementioned studies, which allow students to interact with computers in pairs or small groups, have demonstrated positive learning effects (Johnson & Johnson, 1999). Previous studies have also indicated that peer tutoring may be a useful approach to facilitate students' abilities of helping each other (Webb & Mastergeorge, 2003; Walker, Rummel, & Koedinger, 2011), potentially increasing their mathematical communication. Furthermore, Sorsana (2005) argues that peers' mutual dialogues may benefit students' performance more than a teacher's direct instruction can because students as tutors can instruct their own concepts to peers as tutees. Whereas the tutor learns by doing and teaching, the tutee learns by observing, analyzing, and offering performance-related feedbacks (Topping, 2005; Berghmans, Neckebroeck, Dochy, & Struyven, 2013).

Because playing the role of either tutors or tutees has different learning effects, reciprocal peer tutoring, in which a pair of students interchangeably play both roles (Pigott, Fantuzzo, & Clement, 1986), may be more effective as students mutually explain and convey their opinions, thoughts, and strategies (Brendefur & Frykholm, 2000). For doing so, students have to prepare themselves for instruction, evaluation, and reinforcement, thereby creating mutual assistance and social support (Fantuzzo, King, & Heller, 1992). In this vein, reciprocal roles are intended to promote mutuality in the tutoring process and provide equivalent opportunities. Furthermore, both roles can be engaged in various cognitive and metacognitive activities by using mathematical language (De Backer, Van Keer, & Valcke, 2012). For example, because students need to provide immediate learning information for a partner, they have to disclose their thoughts to each other. Moreover, they have to understand classmates' thought processes as well as the strengths and weaknesses of the proposed explanation, thereby providing additional feedback (Mosston & Ashworth, 2002). Due to questioning, explaining, and self-monitoring of learning, both students may benefit from providing help through teaching each other (King, Staffieri, & Adelgais, 1998).

During reciprocal peer tutoring, tutors usually start from expressing their ideas to peers, so that expressing coherent and organized concepts is a determinant of students' mathematical communication and learning. The reasons are as follows. First, expressing one's concepts is a direct result of students engaging in mathematical communication activities, such as explaining, manipulating different representations, questioning, answering, and correcting others' errors (King, 1998). Second, students can rehearse their knowledge, integrate prior knowledge into new knowledge, and generate new ideas. Third, to produce relevant explanations, students have to prioritize the information and decide which concepts are most related to core topics then rearrange conceptual connections (Chi, Roy, & Hausmann, 2008). Fourth, students may monitor and reflect knowledge building of expressing ideas that help them evaluate the breadth and depth of their own knowledge and improve incorrect or insufficient ideas (Roscoe & Chi, 2007; 2008). Fifth, students should carefully break down examples to many steps and linking them to underlying principles thereby gaining a deeper understanding (Atkinson, Renkl, & Merrill, 2003). Therefore, to train students' mathematical expression, facilitating students' mathematical communication abilities and learning performance is necessary.

Among many strategies for enhancing students' mathematical expression, asking students to generate explanations for others may be a better one (Fiorella & Mayer, 2014). Previous studies have indicated that students who can explain their solution steps of word problems are more successful in transferring knowledge (Aleven & Koedinger, 2002). Cox (1999) also notes that multiple representations can benefit students' expression because higher level of cognitive representation is essential for advanced problem solving skills and communication. Thereby, enhancing students' mathematical communication abilities can be achieved by asking students to integrate different representational forms to solve word problems and to communicate with others. In a sense, the multiple representations are also considered as a communication tool. Based on these reasons, the aim of this study is to explore the students' mathematical teaching material construction effects. The mathematical teaching material used for reciprocal peer tutoring, named math creation, includes multiple representations. The math creation activities emphasize students' mathematical problem solving and communication abilities, which require students to clarify how and why they know, and what they already know about the problems (Banger-Drowns, Hurley, & Wilkinson, 2004) via drawing representations and writing explanations. In addition, previous studies have indicated that computer-supported reciprocal peer tutoring may be an adequate and efficient learning approach to improve students' mathematical communication ability. Therefore, the research questions of this study are: (1) Does a reciprocal peer-tutoring-enhanced mathematical communication (RPTMC) activity improve students' mathematical communication ability? (2) Does students' math creation in RPTMC enhance their performance in expressing their respective mathematical concepts?

RPTMC activity flow

To improve students' mathematical communication ability, a reciprocal peer-tutoring-enhanced mathematical communication activity flow was designed and implemented. Figure 1 illustrates the details of the activity flow. Before this activity, the researchers described the purpose and procedure of RPTMC activity and system functions to both teacher and students. Later on, every two students were paired as a mathematical communication group. The learning activity involved four sub-activities: creating, reciprocal peer tutoring, revising, and staging. The four sub-activities are described as follows.

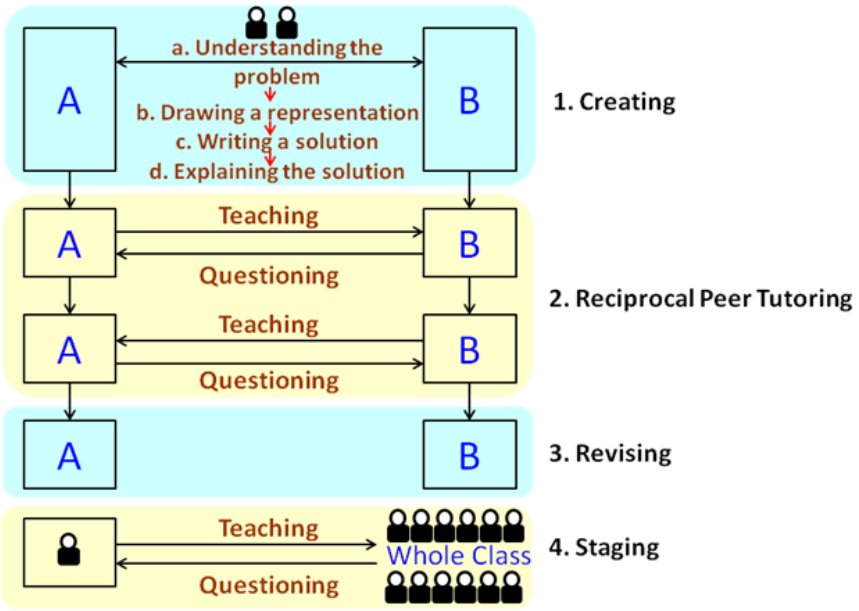


Figure 1. Paired mathematical communication activity flow

Creating

The sub-activity of creating, which required students to prepare tutoring materials, math creations, involved four steps: understanding the problem, drawing a representation, writing a solution, and explaining the solution. These

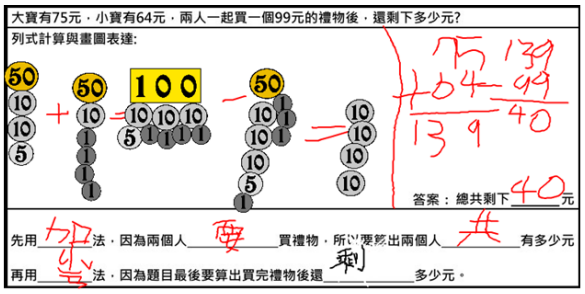
steps were designed according to Polya's findings (1973) about problem solving, i.e., understanding the problem, devising a plan, carrying out the plan, and looking back. The following steps were implemented in the Sketch Board Zone (Figure 3) to assist students in developing a math creation:

- Understanding the problem: Students read the word problem on their own tablet PCs and discussed the solution with their peers to understand the conditions given and the problem asked.
- Drawing a representation: Students used words, symbols, models, and manipulative materials as their mathematical representations to devise a plan as well as to convey their ideas and communicate information.
- Writing a solution: Students wrote their mathematical equations for solving the problem.
- Explaining the solution: Students reflected on how and why they had solved the problem and explained their solution in writing. Because students may need guidance in learning how to express their mathematical concepts before they could write a complete sentence explaining their solutions, a text-based scaffold was provided (see Figure 2).

Reciprocal peer tutoring

In the second sub-activity, paired students sat together to reciprocally teach their math creations. One student, who played the role of a tutor, taught his/her peer why and how to solve the word problem by displaying his/her math creation in the Sharing Zone (Figure 3), while the other student, who played a tutee, received instruction with the tutor's math creation on his/her own tablet PC. Subsequently, the tutee had to ask the tutor questions about the solution strategy. The paired students then switched roles and continued the sub-activity. Figure 2 shows examples of the paired students' math creations.

Student A



Student B

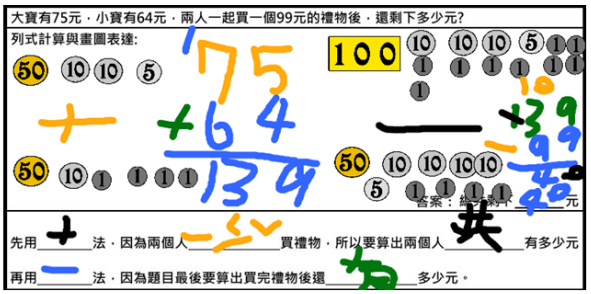


Figure 2. Example of paired students' math creations

Revising

In this sub-activity, the students had to revise their math creations based on peer feedback in the previous sub-activity for improving the correctness and clarity of their own math creations. Revising also served as a time for self-refection and preparation for the next sub-activity of staging. Meanwhile, the teacher monitored the students' math creations and helped their revision.

Staging

Finally, the teacher encouraged the students in each group to display their math creations to the whole class. As their practice in the second sub-activity, they had to explain their solutions with their representations to the audiences. Then they had to answer questions asked by the audiences. In the end, the teacher used students' works to demonstrate how to explain the mathematical concepts and to clarify some mistakes made by students for preventing similar ones next time. Moreover, the teacher may ask some relevant questions to promote students' thinking for communicating their own mathematical concepts and thinking with others.

System design

To explore students' mathematical communication abilities improvement in a one-to-one learning environment (Chan et al., 2006), i.e., one student to one computer, a mathematical communication system is developed. This system provides representational tools for students to construct math creations effectively in a computer-based mathematics learning environment (Hwang, Su, Huang, & Dong, 2009; Sedig, 2008) and express personal mathematical concepts as the math creation of teaching materials to tutor each other. The RPTMC includes two major functions: Sketch Board (Figure 3) designed for constructing mathematics creations, and Sharing Zone for displaying and sharing students' mathematics creations.

Sketch board

The Sketch Board is designed for students to express and explain mathematical concepts by drawing and writing. In the zone, students may complete a math creation individually in the sub-activity of creating. The system also provides a mathematical component library, which includes coins, building blocks, number lines, etc., for students to construct their math creations in order to concretely illustrate the problem-solving procedures and communicate mathematical concepts with others.

Additionally, the Sketch Board has a text-based scaffolding to help students get familiar with solution explanation. In the initial activities, students were scaffolded to think relevant mathematical concepts by completing the keywords of word problems. Students may thus learn how to explain their mathematical ideas to others by imitating similar explanatory patterns. Students started from filling in one-blank sentence, later on filling in more blanks, and after several activities they should write a complete sentence by using conjunction scaffolding. The format of the conjunction scaffolding likes: "[First, I used] one calculating method, [because] (I) wanted to... [Then, I used] another calculating method, [because]..."

Figure 3. Sketch-Board Zone in RPTMC system

Sharing zone

The Sharing Zone is designed to facilitate students to display their math creations easily and instantly. Teachers and students can select a classmate's ID on their own, and then the students' math creations are displayed on the screen for them to observe. This function assists students in taking advantage of their own tablet PCs to view each other's work, enabling them to tutor each other reciprocally for training mathematical oral communication and to get more perspectives and suggestions. This zone also helps students learn various mathematical expressions from others' math creations and return to modify theirs. In addition, through the Sharing Zone, teachers are able to monitor students' progress, analyze students' problems, and examine their knowledge status for subsequently providing real-time feedback and recommend revisions.

Besides, the Sharing Zone provides the enquiring and testing questions sheet revised from Mason (2010) to prompt students' mathematical thought and ask questions to help peers mutually examine the correctness of their mathematical representation, solution, and solution explanation. Enquiring questions may guide learners to understand and ask questions about every meaning from several divided problem-solving steps of representation and solution. If students do not have their own questions, they can adopt the question sheet to participate in the reciprocal peer tutoring. As well, the enquiring questions can also be regarded as hints to remind students what and how to express their individual concepts to a peer or to others. Furthermore, the testing questions sheet is used for students to ask advanced solution explanation in case that the explanations of peers are unclear or doubtable (see Table 1).

The following example demonstrates how the students participate in the RPTMC activity. At the Creating Stage listed in Figure 1, each student receives a mathematical word problem on the Sketch Board delivered by the RPTMC system. At this stage, the students start to create their own teaching materials, try to understand the problem, then construct their math creations (representation, solution, and solution explanation). The student also has a text-based scaffolding to help him/her develop solution explanation. Once the students have their own math creations, the students practise reciprocal peer tutoring via the Sharing Zone. At the stage of reciprocal peer tutoring, the student who plays the tutor introduces his/her solution processes to the student who plays the tutee. The tutee then ask the tutor questions either based on the enquiring questions listed in Table 1 or the questions generated by himself/herself. After the tutor answers the questions proposed by the tutee, the tutee is assigned a set of testing questions to check the correction of the math creations constructed by the tutor. The paired students exchange their roles once this sub-activity is completed. The following is the revision stage, in which the students revise their math creations according the feedbacks generated in above sub-activities. At the final stage, staging, the teacher selects one or two students from each group to illustrate their math creations for the whole class. And then, the teacher gives some comments to those students' math creations and makes a conclusion.

Table 1. Two question sheets embedded in the sharing zone

Enquiring questions	Testing questions
(1) What does this problem ask for? (2) How do you illustrate six boxes of milk, with each box having twelve bottles of milk? (3) How do you illustrate the milk drunk? (4) How do you illustrate the milk left? (5) What does each calculation mean?	(1) If there is something unclear or incorrect in the Instructor's illustration, please question him/her. (2) Is his/her solution correct? If there is something wrong, please tell him/her the correct solutions. (3) Is his/her solution explanation correct? If there is something wrong, please tell him/her how to modify it.

Evaluation

Setting

For answering the research questions, an experiment was conducted. Fifty-one second-graders (aged eight to nine years) from two classes at a primary school in Taiwan were involved. The school had established a one-to-one learning environment in their math program. These students learned math by using one-to-one technology in every formal math course since they were in the first grade; thus, they had sufficient computer operation abilities to participate in the RPTMC activity. Specifically, the activities were conducted 13 times over an entire semester. Each time took 80 minutes each week. In this study, one class was assigned to be the experimental group (25 students, 13 boys and 12 girls), and the other one served as the control group (26 students, 14 boys and 12 girls).

The control variable was their daily learning approach. In other words, both groups had the same mathematics learning time in the same one-to-one self-learning mathematics environment. However, the control group practised mathematics by teacher-led instruction for solving various word problems, while the experimental group participated in the RPTMC activity to solve related word problems chosen by the teacher and researchers. For evaluating students' mathematical communication ability, the pretest and posttest were conducted before and after the experiment, respectively. Each test took 40 minutes.

Pretest and posttest: Mathematical communication ability assessments

For the first research question, the mathematical communication assessment was used to assess the students' mathematical communication abilities in terms of three sub-abilities: expressing their respective mathematical concepts, understanding others' mathematical equations, and comprehending others' mathematical thought (Lin & Lee, 2004). The assessment was consisted of multi-step word problems, including continuous addition, mixed addition and subtraction, mixed addition and multiplication, and mixed subtraction and multiplication. The pretest and posttest used the parallel problems all within the 2nd-grade mathematical curriculum (see Figure 4). These problems were collaboratively designed and developed by two educational technology experts and an elementary school teacher who has ten years teaching experience. Each question represented one mathematical communication sub-ability, and each sub-ability included two to three criteria to test different evaluative approaches.

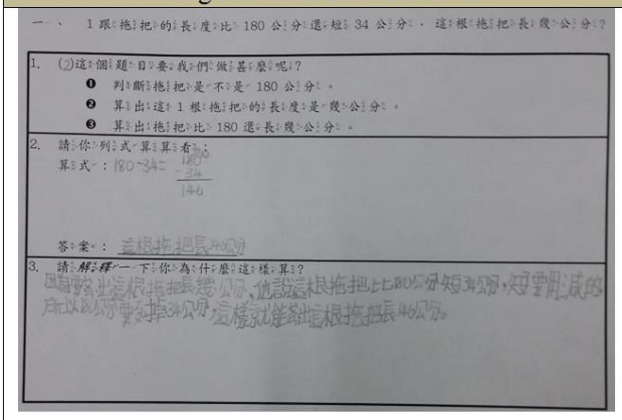
Original assessment material	Translated assessment material
	1. Understand the meaning of a mathematics problem: What does the problem ask for? 2. Use mathematical symbols to solve the problem: Please list the equation to solve the problem. 3. Apply mathematics language and representations to explain mathematical concepts: Please explain why you calculated in that way.

Figure 4. Example of mathematical communication assessment

Evaluation criteria

Table 2 lists the detailed evaluation criteria for mathematical communication abilities (Lin & Lee, 2004). The criteria and scores were devised through discussions with three educational assessment experts and seven schoolteachers.

Table 2. Evaluation criteria for mathematical communication abilities

Sub-ability	Criteria	
Express individual mathematical concepts (8 points)	(1)	Understand the meaning of a mathematics problem (multiple choice, 1 point)
	(2)	Use mathematical symbols to solve the problem (word problem, 2 points)
	(3)	Apply mathematics language and representations to explain mathematical concepts (explanation problem, 5 points)
Comprehend others' mathematical equations (6 points)	(1)	Understand others' mathematical equations and evaluate the correctness (true/false question, 1 point)
	(2)	Provide meaningful explanations for correct equations or explain reasons for incorrect equations to others (explanation problem, 5 points)
Comprehend others' mathematical thought (6 points)	(1)	Realize others' mathematical thought and evaluate the correctness (true/false question, 1 point)
	(2)	Use mathematical language to transform others' mathematical thought into mathematical equations (if true) or ask meaningful questions and explain reasons for the incorrect thought of others (if false) (explanation problem, 5 points)

To ensure the reliability of the assessment, two raters evaluated the assessment independently. The inter-rater reliability of the pretest was 0.912, $p < .05$, and that of the posttest was 0.905, $p < .05$.

Results

Table 3 presents the averages and standard deviations of the pretests and posttests of both groups. A *t* test showed no significant differences between the pretest scores of the two groups [$t(49) = .48$, $SE = 1.23$, $p = .80$], indicating both groups had equivalent mathematical communication ability. Besides, the pretest scores of both groups were only around half of the total score, suggesting that students needed more trainings to enhance their mathematical communication abilities, especially those for comprehending others' mathematical thought.

Table 3. Students' performance levels on the pretest and posttest

Mathematical communication ability		Experimental group (n = 25)		Control group (n = 26)	
		Pretest	Posttest	Pretest	Posttest
Express students' respective concepts (8 points)	Mean	4.28	7.16	3.96	5.81
	SD	1.34	1.77	1.25	1.65
Understand others' mathematical equations (6 points)	Mean	3.00	4.12	3.65	1.77
	SD	2.55	2.03	2.86	1.39
Comprehend others' mathematical thought (6 points)	Mean	2.44	5.20	2.69	3.00
	SD	1.83	1.58	1.49	1.92
Total	Mean	9.72	16.48	10.30	10.58
	SD	4.62	4.29	4.18	2.94

A two-way ANOVA ($\alpha = 0.05$ significance level) was performed to analyze the effect of RPTMC activity on the total scores of the mathematical communication assessments. The variance homogeneity tests between group linearity were not significant ($p = .40 > .05$). The results revealed a significant interaction between the groups and the pre/posttest [$F(1, 49) = 23.76$, $MSE = 271.80$, $p = .00$; $\eta^2 = .15$]. Further analysis indicated that the experimental group showed significant improvement in total score [$t(24) = -7.64$, $SE = .89$, $p = .00$], whereas the control group showed no significant improvement [$t(25) = -.27$, $SE = 1.00$, $p = .79$]. In other words, students engaged in RPTMC activities may develop mathematical communication abilities superior to those who received one-to-one self-paced learning and teacher-led instruction. The reason may be that the RPTMC activity could promote student use of mathematical language for expressing mathematical concepts and sharing math creations in the Sharing Zone; thus, the students may learn various forms of expression for communicating their solutions with tutored peers. In addition, when monitoring the students' performance of math creation, the teacher asked some students to provide more detailed explanations of their solutions and drawings, and thereby develop their mathematical communication abilities after several activities. Contrarily, the control group had fewer opportunities to express their mathematical ideas about solving their problems, which resulted in lower performance on the posttest.

In addition, the experimental group showed a significant improvement on the sub-abilities of expressing their respective mathematical concepts [$t(24) = -7.96$, $SE = .36$, $p < .01$], understanding others' mathematical equations [$t(24) = -2.21$, $SE = .51$, $p < .05$], and also comprehending others' mathematical thought [$t(24) = -6.16$, $SE = .45$, $p < .001$]. The results suggested that all three sub-abilities were fostered by training in the RPTMC activities. In other words, sufficient practices on finding solutions and explaining them through writing/drawing and verbal forms may assist students in expressing their own mathematical concepts and understanding others' mathematical thought.

Math creations: Expressing individual mathematical concepts

For the second question, this study further analyzed the performance of students' math creations in the experimental group. Besides, the researchers also interview the class teacher and students about students' learning differences and difficulties before and after the RPTMC activities.

Evaluation criteria

The scores of expressing their respective mathematical ideas were calculated according to the criteria in Table 4, in which the correctness and completeness of their creations were considered. Each criterion was scored as five points. The criteria were designed to evaluate how well the students (1) applied various mathematical representations for explaining their respective mathematical concepts, (2) used mathematical symbols to solve problems, and (3) explained the solutions critically.

To observe the development of the students' math creations, the 13 weeks of math creations were divided into three stages according to the mathematical learning content: The initial stage (weeks 1 to 2), in which two-digit subtraction and two-digit by one-digit multiplication were learned; the middle stage (weeks 3 to 7), in which continuous addition, continuous subtraction, and mix of addition and subtraction; and the final stage (weeks 8 to 13), in which mix of addition, subtraction, and subtraction were learned.

Table 4. Evaluation criteria for the math creations

Score	Mathematical representation	Solution	Solution explanation
Incorrect or not related (1 point)	None or drawing something hard to understood	Incorrect equation	Explaining incorrectly
Correct but without a critical concept (2 points)	Drawing objects without relevant marks	Only part of a correct equation	Explaining irrelevantly
Correct but focused on the calculation (3 points)	Drawing objects as an equation without their amounts or meanings	Correct equation with an incorrect calculation	Explaining only the calculation
Correct but incomplete (4 points)	Drawing objects as an equation with their amounts or meanings	Correct equation without an answer or with an incorrect answer	Explaining correctly without essential concepts
Correct and complete (5 points)	Drawing spatial relations between the objects of the problem and the solution strategy	Correct equation with a correct answer	Fully elaborating solution strategy with spatial relations among the various objects of the problem

Results

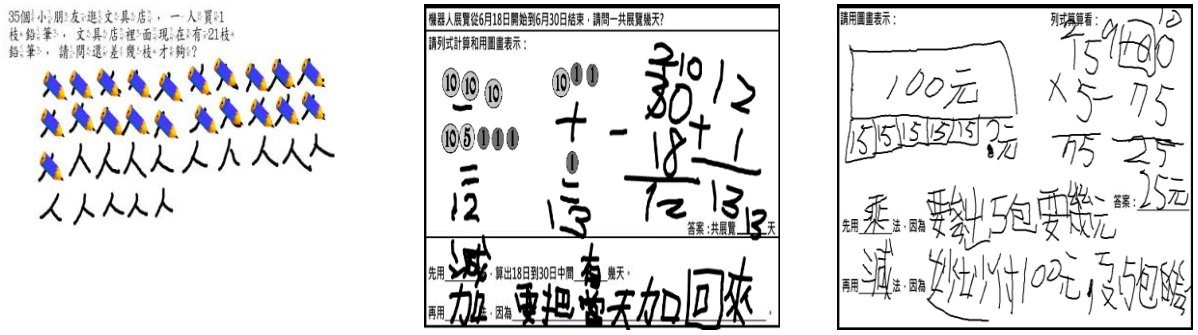
The average scores for the quality of the students' creations at the three stages are listed in Table 5. Besides, to illustrate the math creations, a typical example of one's math creations selected based on the average scores of the three stages is shown in Figure 5.

Table 5. Three stages of expressing students' respective mathematical ideas

Stage	Mathematical representation	Solution	Solution explanation
Initial	1.86 (1.54)	3.66 (2.14)	—
Middle	2.04 (1.01)	4.76 (0.90)	2.39 (0.91)
Final	3.29 (1.60)	4.94 (0.44)	3.06 (1.66)

At the initial stage, the students were arranged to get familiar with the system and the rules for math creation, so they did not need to write the solution explanation. Half of the math creations were basically correct but without essential concepts. Specifically, they used various math components as mathematical representation for expressing the amounts of the variables in the problems. Some students' mathematical representations were not comprehensible, whereas the other students' presented only partial equations for the solutions. These creations showed that most students lacked sufficient understanding to transform the details of the problem into their representations; therefore they could not apply appropriate mathematical representations to express their mathematical concepts at this stage. Besides, in the interview some students regarded the mathematical representation as the solution, and thereby they did not write the solutions.

At the middle stage, most students started to use coins or blocks provided by the system to represent the equations for their solutions and calculative processes. Specifically, their mathematical representations showed an amount and order similar to their equations instead of solution strategies at this stage. In addition, when explaining solutions with scaffolds, many students did not know the answers and left the blanks, thereby resulting in a relatively low average score. By contrast, the average solution score increased, because the students were more familiar with the differences between mathematical representation and the solution, and recognized the mathematics problem meaning. Hence, they were more capable of using correct mathematical symbols to complete the solutions.



Initial stage Middle stage Final stage

Figure 5. Example of students' math creations at the three stages

At the final stage, the teacher reflected that "students began to use various explanatory methods and also examined the process and reasoned one step after another," indicating that the students attempted to apply various mathematical representations and reasons for expressing their solution ideas and could think how to solve a word problem logically. The students started to draw squares or circles with numbers as their mathematical representations, and then their scores increased. As a result, from tutors' mathematical representations, tutees could better observe the solution strategies and the relationship of every condition to the assigned problem. Regarding the solutions, although they were assigned more difficult problems, the average score increased. In addition, the average of the solutions explanations also slightly increased because they were requested to write a complete explanation. However, most students merely wrote arithmetic operations instead of sentence. There were seldom students expressed their reasons for representing the meaning of each number. The results indicated that the students became more capable of using mathematical representations and make equations, but they still needed more practice in using mathematical language for explaining their solutions.

In sum, the students exhibited considerable difficulty in drawing their mathematical representations and solving problems at the initial stage. Meanwhile, the students became increasingly capable of constructing their math creations as mathematical communication artifacts at the middle stage, and at the final stage, their math creations demonstrated more sophisticated representations and explanations.

Conclusions

Mathematical communication emphasizes people's interactions and exchanges of mathematical ideas, which is a crucial ability for students in expressing their respective mathematical concepts, comprehending and evaluating other students' mathematical equations and thoughts. Therefore, this study aims at enhancing pupils' mathematical communication abilities. To assist students in constructing math creations for reciprocally tutoring their classmates (e.g., to convey mathematical concepts and understand peers' mathematical ideas), an RPTMC system was designed and implemented. For evaluating mathematical communication ability, both math creation and mathematical communication assessments were applied.

The evaluation showed that the experimental group students' mathematical communication abilities had improved. More specifically, the RPTMC activity could facilitate pupils' three sub-abilities: (1) to express their respective mathematical concepts, (2) to understand others' mathematical equations, and (3) to comprehend others' mathematical thought. The findings provided insights into the manner of their progress. Regarding comprehending others' mathematical thought, the students improved significantly in the assessments after sufficient RPTMC

activities. At the pretest stage, because most students were unable to communicate math with others, they wrote only equations or simply repeated the statements of a problem. However, when playing the role of tutees in the reciprocal peer tutoring, they had to use appropriate mathematical language to transform their partners' mathematical thought into mathematical equations. Such a collaborative and shared activity may allow them to realize peers' mathematical thought, evaluate the correctness, and further provide meaningful questions and explanations for the peer's incorrect thought. The improvement on the assessment indicated those students' experiences of the RPTMC activity may enable them to transfer other's knowledge to self-expression for more complete and understandable creations. Therefore, in the posttest, most students understood the meaning of mathematical problems and knew which mathematical symbols should be used in solving their math problems. Most importantly, their thoughts were sharpened by connecting to various presentation techniques to achieve the effect of mathematical communication.

However, although the students could express their mathematical solutions and explain others' mathematical equations and thoughts with complete sentence in their posttest, most students did not write complete sentences to explain their solutions in their math creations. Such phenomena reflected that students might have learned how to express their mathematical concepts, but some factors still limited their performance in the math creations. For instance, students were used to typing rather than handwriting words on tablet PCs. Even if they knew how to explain their solutions, they did not do so. In addition, as some math creations without explanation were shown in the Sharing Zone, more and more students imitated such creations and eventually formed an improper classroom atmosphere. This fact revealed that the Sharing Zone can facilitate the students' mathematical communication, but some students might misuse the function. Thereby when an instructor finds learning problems spread by the learning system, the instructor's immediate correction is necessary.

Regarding the sub-ability effects of understanding other students' mathematical equations, the experiment demonstrated significant learning gains, possibly because the control group was unfamiliar with understanding other students' mathematical equations. In other words, teacher-led instruction with one-to-one self-learning lacked opportunities for mathematical communication with their peers possibly prevents them from understanding other's equations.

Overall, the RPTMC activity not only improved students' mathematical communication ability but also provided instructors and system designers with the developmental process of mathematical communication ability. By doing so, instructors are more capable of analyzing the quality and improvement of students' mathematical communication ability (i.e., depicting the quantitative relationships of mathematical problems, completing mathematical representation, and explaining solutions). By using the Sharing Zone function of the RPTMC system, instructors could have the students' learning statuses instantly. However, system designers still need to improve the system by developing categorization or grouping functions for instructors to assess and monitor the students' learning performance of each group. In addition, once students complete the RPTMC activity, the system should provide students with the opportunity to exchange suggestions for mutual math creations, thereby enhancing mathematical communication further. The system and learning approach demonstrated in this study is only one of the methods for stimulating mathematical communication abilities. Further class computer-supported reciprocal peer tutoring methods investigations are needed for realizing computer-supported reciprocal peer tutoring learning approaches in class.

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